MATH 256 Section 202
W2
Midterm Exam 1
February 1, 2017
Time Limit: 45 Minutes

Last Name: ____________________

First Name: ____________________

Student #: ____________________

This exam contains 6 pages (including this cover page) and 4 problems. Enter all requested information on the top of this page.

The following rules apply:

- You may **not** use your books, notes, or any calculator on this exam.
- Unless a question asks you to state, or write down the answer, you must **show all your working**. There is credit given for using the correct method to solve each problem. **Unsupported final answers will not receive full credit.**
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering is very difficult to mark and might receive reduced credit if your argument is not clear.
- If you need more space, use the back of the pages.

Do not write in the table to the right.

Problem	Points	Score
1	4	
2	9	
3	6	
4	13	
Total:	32	

Do not open the exam until instructed to do so.

1. For each of the following differential equations for the function $y(x)$, state the order of the differential equation and state whether it is linear or nonlinear.

   (a) (2 points)

$$\frac{\mathrm{d}y}{\mathrm{d}x} + xy = x^2$$

   (b) (2 points)

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + \frac{\mathrm{d}y}{\mathrm{d}x} + x^3y = e^y$$

2. (a) (3 points) What is the integrating factor for the following differential equation for $y(x)$?

$$\tan(x)\frac{\mathrm{d}y}{\mathrm{d}x} + y = f(x)$$

Hint:

$$\int \cot(x)\,\mathrm{d}x = \ln(\sin(x)) + C \text{ where C is an arbitrary constant.}$$

(b) (6 points) What is the general solution to the following differential equation?

$$\frac{\mathrm{d}y}{\mathrm{d}x} + 2xy = 2x$$

3. (6 points) Solve the following differential equation for $y(x)$ with boundary condition $y(0) = 1$. Leave your answer as an implicit equation involving y and x.

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{e^{3x}}{y-2}$$

4. Consider the following differential equation for $y(x)$:

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 2\frac{\mathrm{d}y}{\mathrm{d}x} + y = f(x).$$

(a) (3 points) What is the homogeneous solution $y_h(x)$?

(b) (5 points) For each of the following functions $f(x)$, *write down* the form of the particular solution $y_p(x)$ that will solve the differential equation considered in part (a). You do not need to solve for the unknown coefficients.

i. $f(x) = 2x + 3$

ii. $f(x) = 5\sin(2x)$

iii. $f(x) = 6e^{2x}$

iv. $f(x) = (3x + 2)e^{x}$

v. $f(x) = 4e^{-x}$

(c) (5 points) What is the general solution to the following differential equation for $y(x)$?

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\frac{\mathrm{d}y}{\mathrm{d}x} + y = 2x + 3$$

Hint: This is the same equation considered in parts (a) and (b) with the function f(x) given in (b) part (i).